A NEW MICROSCOPIC METHOD FOR MEASURING THE VISCOSITY OF A LIQUID.

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Although the methods and apparatus for measuring the viscosity of a liquid found in literature are numerous in number and differ in principle, we can classify them into following categories:

(1) Those based on the measurement of the velocity of the liquid efflux from a horizontal or a vertical capillary tube.

(2) Those based on the measurement of the motion of a spherical or a cylindrical body in the liquid.

(3) Those based on the measurement of the angle of rotation of one cylinder dipped co-aixally in another hollow cylinder filled with the liquid.

(4) Those based on the measurement of the rotatory oscillation of a sphere, a cylinder, or a circular disk in the liquid.

(5) Other special methods.(1)

The new method of measurement proposed here differs in principle from those cited above and, having several characteristic features, is very useful, especially for the measurement of viscosity of viscous liquids, e.g. colloidal solutions of high concentration.

Theoretical.

The principle of this new method of measurement is quite simple. The liquid to be measured is made to flow under a definite pressure in a capillary tube of definite radius and length. The velocity of flow on the axis of the capillary tube is measured directly under a microscope and thus the absolute viscosity is determined. The theoretical formula is derived in the same way as in the case of Poisuille's law. When a

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(1) A. Lampel, *Sitzb. Akad. Wiss. Wien,* (2a), **93** (1886), 291. H. Helmholtz and G. Piotrwski, *Sitzb. Akad. Wiss. Wien,* (2a), **40** (1868), 607. R. Ladenburg, *Ann. Physik,* (4), **27** (1908), 157. K. Mützel, *Wied. Ann.,* **43** (1891), 15. Subrahmaniam. *Indian J. Physics,* **1** (1926), 267. G. Stokes, *Phil. Mag.,* (4), **1** (1882), 337. F. Watson, *Phys. Rev.,* **15** (1902), 20. C. E. Guye and S. Mintz, *Arch. sci. phys. nat.,* (4), **26** (1908), 136. Gümbel, *Z. tech. Physik,* **1** (1920), 72. R. O. Boswall, *Phil. Mag.,* (7), **3** (1927), 994. G. Richardson, *J. Scient. Instrument,* **6** (1927), 337. H. A. Wilson, *Proc. Cambridge Phil. Soc.,* **10** (1900), 25. R. Z. Fürth, *Z. Physik,* **60** (1903), 313.

liquid flows without slip or turbulence in a capillary tube, the following relation is given:

$$v = \frac{P}{4l\eta}(R^2-r^2) \qquad (1),$$

in which, v is the velocity of the flow at the distance r from the axis, l the length, and R the radius, of the capillary tube, P the difference of pressure at the two ends of the capillary tube, and η the viscosity of the liquid. Putting $r = 0$, we have v_0, the velocity on the axis of the tube:

$$v_0 = \frac{PR^2}{4l\eta} \quad (2), \qquad \text{hence} \qquad \eta = \frac{PR^2}{4lv_0} \quad (3).$$

As P, R, and l are known, we can determine the value η, if we can measure v_0. In practice, the measurement of P, R, and l is so simple that no explanation will be needed. It would be necessary to discuss how we can measure v_0, or whether v_0 is a measurable quantity.

I. v_0, a Measurable Value. Let us assume the viscosity of the liquid to be 10 (in absolute unit), the order of the viscosity of glycerine at ordinary temperature, and also, the radius of the capillary tube to be 0.05 cm. and its length 10 cm. Putting these values in the equation (2), we have $v_0 = 0.000625P$. Suppose a pressure is given by a water column, and let the height of the column be h, and we have $v_0 = 0.0613h$. The relations between v_0 and h are shown in Table 1. From this Table, we see that, under the pressure of the water column ranging from 1/100 mm. to 100 cm. in height, the velocity may be measured with the aid of a microscope and an ocular micrometer. For, as the displacement in the microscopic field is v_0t (t being the time required for the displacement), if we assume t=1000 sec., the measurement of the minimum value of v_0 in the table is possible. For a larger value of v_0, it is possible to measure with the same accuracy up to the value ca. h=0.1, if we take t smaller than above. In short, it may be said that, as v_0 is a measurable value even when radius R is 0.05 cm., we can always bring the value v_0 within the possible range of measurement by regulating the radius, the length,

Table 1.

h (cm.)	v (cm./sec.)
0.001	0.0000613
0.01	0.000613
0.1	0.00613
1	0.0613
10	0.613
100	6.13

and the pressure. So, it may be concluded that v_0 is a value measurable in the microscopic method.

II. The Range η can be Measured. Putting reasonable values for P,R,l in equation (3) let us try to determine the range where the measurement of η is possible.

R	(minimum value) 0.01 cm.	(maximum value) 0.5 cm.
l	5 cm.	20 cm.
P	1 cm. (water column)	150 cm. (mercury column)
v_0	0.00005 cm./sec.	0.005 cm./sec.

Measuring under these conditions with the aid of a stop watch, we have, within the accuracy of 0.001, the following numbers for η:

minimum value 2.4×10^{-1} (absolute unit), maximum value 5.0×10^{8} (absolute unit).

Therefore, the range is 10^9. For trial, if we calculate the range measurable with an Ostwald type viscosimeter, under the similar conditions, assuming the volume of liquid to be 5 c.c., and the time of flow 50 sec. (minimum), and 1000 sec. (maximum), we have:

minimum value 3.9×10^{-6} (absolute unit), maximum value 1.9×10^{6} (absolute unit).

In the method of falling ball, let the conditions be:

radius of the ball	(minimum value) 0.1 cm.	(maximum value) 0.5 cm.
time of fall	100 sec.	1000 sec.
density of ball	1.06	16.6

Furthermore, let the density of the liquid be 1 and the distance of the fall 10 cm., and we have, with the aid of Ladenburg's equation of correction:

minimum value 1.4×10^{0} (absolute unit), maximum value 6.5×10^{6} (absolute unit).

In practice, it is difficult to get perfectly spherical balls with different densities. Therefore, if we use the steel balls for ball-bearing on the market, we have the following numbers:

minimum value 16×10^{2} (absolute unit), maximum value 2.8×10^{6} (absolute unit).

Comparing these results, we see that the ordinary capillary viscosimeter is suitable for liquids of smaller viscosity, while the microscopic method is more useful for liquids of higher viscosity, and that the possible range of measurement is the largest in the latter.

III. Practical Method of Measuring v_0. A practical method of measurement in this microscopic method is as follows: first, fine solid particles are mixed with, and uniformly dispersed in, the liquid, and the liquid is put in a capillary tube. Then, with the aid of a microscope, we select any one particle among the particles which lie on the axis of the capillary tube. With this particle for an indicator, the velocity of the translation of the liquid is determined with the aid of an ocular micrometer and a stop watch. Therefore, it is necessary that the properties, the quantity to be added, and the radii of these particles should be examined.

(a) *Selection of Particles.* It is necessary that we should use the particles of a solid substance which is perfectly free from chemical reaction with, or from being dissolved by, the liquid to be investigated. For example, powder of carbon, platinum, silica, glass, Al_2O_3, SiC, etc. are suitable for this purpose. Though adsorption by the particles may be supposed to occur, this is negligible as the quantity to be added is very small. Of course, there is no need of taking this into consideration when the sample liquid is pure. It may be added that the particles must be free from swelling in the liquid.

(b) *Quantity of Particles to be Added.* The effect of the particles added on the viscosity of the liquid is as follows. Suppose that η, the viscosity of the sample, changes to η', on account of the addition of the particles, and that the measurements are done with the same capillary tube and under the same pressure, we have, by the equation (3),

$$\eta = \frac{R^2}{4lv_0} \qquad (4), \qquad\qquad \eta' = \frac{R^2}{4lv_0'} \qquad (5).$$

As the relation given by Einstein is applicable between the viscosity of the dispersed system and that of the medium, we have:

$$\eta' = \eta\,(1+2.5\,\varphi),$$

where φ is the value of the total volume the particles occupy, divided by the whole volume of the dispersed system. From (4), (5), and (6),

$$v_0 = (1+2.5\varphi)\,v_0', \qquad \text{hence} \qquad v_0' = v_0\left(\frac{1}{1+2.5\varphi}\right);$$

$$\text{therefore,} \qquad v_0' = v_0(1-2.5\varphi), \qquad \frac{v_0'-v}{v_0} = -2.5\varphi, \qquad \frac{\Delta v_0}{v_0} = -2.5\varphi.$$

Let m be the quantity of the fine particles in 100 c.c. of the dispersed system, and ρ the density of the particle, then

$$\frac{\varDelta v_0}{v_0} = -0.025 \frac{m}{\rho} .$$

Assuming $m = 0.01$ g., we have Table 2.

Table 2.

ρ	$\Delta v_0/v_0$
0.5	0.0005
1	0.00025
2	0.00013
3	0.00008
4	0.00006
5	0.00005

As has already been noted, if the measurements of v_0 are done, in the case of $m = 0.01$ g., in the accuracy of 1/1000, all values of Table 2 becomes already less than the accuracy. Therefore, the velocity of the particle may be taken as the velocity of the liquid when the latter contains no particle. If we assume $\rho = 3$, the value $m = 0.1$ g. become within the range of the experimental errors. So, the use of the particle up to this value may well be admitted.

(c) *Radius and Number of the Particles.* The particle must be large enough to be an objective in the microscopic field, and the number of the particles must be sufficient so that the selection of the particle on the axis of the capillary tube may be done with ease. If we assume that one cubic millimetre of liquid contains 100 particles, the particle must be present in the probability of 10 particles on one millimetre of the axis. In order to obtain distribution of this density the radius of the particle ought to be

$$r = \sqrt[3]{\frac{3}{4} \frac{m}{\pi \rho n}}$$

where n is the number of particles in 1 cm.^{3}; r radius of the particle, m total mass of the particles, and ρ the density. Let $m = 0.001$ g., $\rho = 1$, $n = 10$, then we have $r = 0.0029$ cm. Therefore, it is sufficient, if we add the particles of radius of 0.03 mm. at the rate of 0.01 g. per 100 c.c. Moreover, this will also satisfy the conditions of the paragraph (b). In practice, as the selection of the objective particle can be done in the state of flow, the number of the particles may be far less than the rate just given.

(d) *Distance of Sedimentation Occurring during the Time of Measurement.* It may happen that, the particle, as the result of sedimentation due to gravity, escapes away from the focus of the microscope while being observed. This point may here be discussed. The distance of sedimentation of a sphere, whose radius is r and density ρ_2, in a liquid with viscosity η and density ρ_1, will be given by the Stokes' law:

$$x = \frac{2}{9}\frac{r^2}{\eta}(\rho_2-\rho_1)gt \;.$$

If we assume the particle to be that of carborundum in water, then the distance x will be equal to 0.014 cm. for the time interval of $t = 1000$ sec.; and if $r = 0.0001$ cm., we have $x = 0.0003$ cm. Therefore, in this case, it is necessary to use the particle of $r = 0.0001$ cm., that is, 1 μ in radius. If the liquid has the density of the degree of glycerine, the particle may be of the radius of about 0.01 cm. With the liquid of higher viscosity, it is almost unnecessary to take this matter into consideration. From these investigations, it may be concluded that the microscopic method of measuring the viscosity of a liquid is a proposition not only practical, but also very convenient. Before describing the results of measurement, some distinctive characteristics of this method may be enumerated here:

(1) The measurement can be secured with a small quantity of the sample, only a few c.c. being sufficient.

(2) The possible range of measurement is wide.

(3) The measurement of the density of the sample is not necessary. In general, it is very difficult to measure the density of a viscous fluid. From this point of view, the present method may be said to be very useful.

(4) No special apparatus of measurement is required.

(5) As the presence of anomalous viscosity or a turbulent flow is recognized directly, the normal value of viscosity may be obtained.

Experimental.

I. Apparatus for Measurement. The measuring cell is made of glass with a horizontal capillary of ca. 10 cm. length and a vertical side tube of ca. 5 cm. height (Fig. 1). The schematic figure of the whole arrangement of the apparatus is given in Fig. 2. In the figure, AB is the measuring cell, M a microscope furnished with an ocular micrometer, F a green light filter which is used to get a sharp image of the objective particle in the field, S an incandescent electric lamp of Leitz make, the light source used in microphotography. At middle C of the capillary tube, two small pieces of glass plates are attached with balsam one above and one below. They serve the purpose of keeping the rays from the light source parallel and of securing a sharp outline of the objective particle. D is a three-way stop cock, N a liquid manometer, P and Q water holders which serve to produce pressure, P being fixed, Q movable up and down by a microscrew. E_1 and E_2 are small glass bulbs containing the same liquid as the sample, and serve to prevent the latter from evaporation or concentration, or taking moisture. In the experiments (1) and (2), where glycerine was used as the material for test, the measuring cell was kept at room temperature, but in the experiment (3), the cell was dipped in a thermostat, and the objective of the microscope was covered with a small glass cap K with a plane bottom, and was dipped in the liquid of the thermostat. The liquid used in N, P, and Q, was pure water, but, if necessary, mercury might be used.

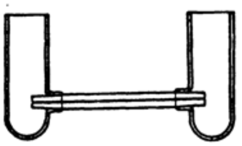


Fig. 1.

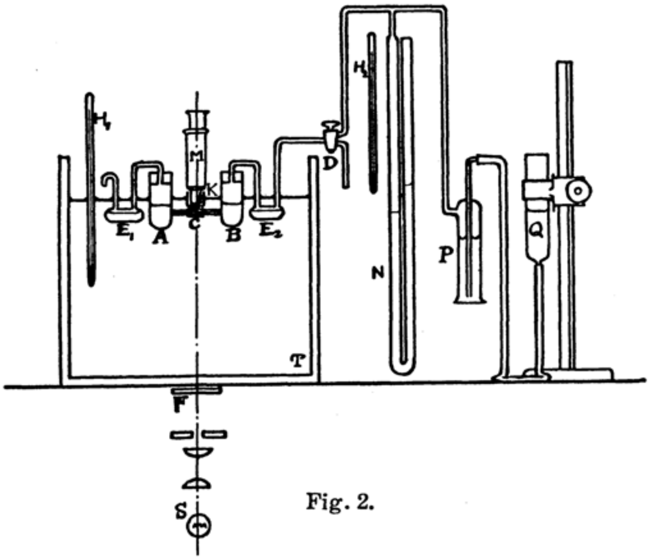


Fig. 2.

II. Procedure. First, the tube for measurement AB is filled with the sample uniformly dispersed with objective particles and then all the connections are completed, as shown in Fig. 2. Then the selection of the objective particle in microscopic field is done. For this purpose, the axis of the capillary tube must be determined in the microscopic field. In order to do this, the ocular micrometer is fixed at right angles to the capillary tube. Then by the image of the walls of tube, the position of the axis of the tube is determined on the scale of the micrometer. Now the ocular is turned, the microscope being fixed, just 90° and the scale of the micrometer is made to coincide with the axis of the tube. Then by raising and lowering the microscope vertically with the focus on the upper and then on the lower limit of the inner wall of the tube, two readings of the microscrew are obtained. The microscope is then fixed at the point just in the middle of these readings. Thus, the focus of the microscope is fixed on the axis of the capillary tube. In this position, the particle seen sharply in the centre of the field may be regarded to be on the axis of the tube. Now opening D, and making the liquid to flow in the capillary tube, a suitable particle is waited for to appear. When the objective particle is selected, the time is measure, with a stop watch, needed for the passage of the particle between two fixed points on the scale of the micrometer. Then the readings of the manometer as well as the thermometer are recorded. It is necessary to repeat the measurement several times under the same pressure until the maximum value of the velocity of the particle is obtained. Then the pressure is changed and measurement is done in the same manner. The measurement must also be done with the flow in the reverse way. The reason for this is that the pressure which causes the liquid to flow is not merely the pressure which is read on the manometer. Besides this, there is also a pressure, which affects the flow, due to the difference of the levels of the liquid in the two side tubes. In many cases, the levels in the side tubes are not equal in height under the condition with the stop-cock D closed. Especially, when the viscosity of the sample is very high, it is difficult that the levels in the two side tubes should become equal in height; and moreover when the surface of the liquid are in a curved state as it may well be, a considerable time will be required for it to regain the normal meniscus. It is necessary, therefore, to determine the value of the pressure due to this difference of the levels. Let us assume that under the positive or negative values of the pressure indicated on the manometer P_1, P_2, P_3, $\ldots . P_n$, the velocities of flow, v_{01}, v_{02}, v_{03}, $\ldots . v_{0n}$, have been obtained respectively. Plotting the diagram with P and v_0 as the coordinate axis, we get straight lines. In general, these straight lines do not pass through the origin. The distance, from the point where the line crosses the P-axis to the origin, indicates exactly the value of the pressure due to the difference of the levels. It may be assumed that this value is practically constant during measurements. A simple calculation shows this. Let the inner radius of the capillary tube be 0.05 cm., that of the side tube be 1.5 cm., and the number of the times of measurement be 10, the time needed for each measurements, the liquid passes only in one direction, and the volume of the liquid translated will be

$V = 1.47 \times 10^{-2}$ cm.^{3}, as $V = \dfrac{\pi R^4 Pt}{8l\eta}$, $R = 0.05$ cm., $P = 10^3$ dyne/cm., $l = 10$ cm., $\eta = 10$, and $t = 600$ sec. Therefore, the change of the levels in the side tubes will be $\Delta h = 2.00 \times 10^{-3}$ cm. which is within the error of the readings of the manometer. In practice, as the liquid travels in both directions, this value becomes smaller; and the pressure due to the difference of levels in the side tubes may be treated as unchanged during measurements. Now, the relation between P and v_0 becomes linear. (Of course, as is indicated in equation (2), it must be so theoretically.) The value P must be the algebraic sum of the two pressures, that is, $P = P_1 + P_2$, where P_1 is the value indicated in manometer, and P_2 is the value due to the difference of the levels. As regards the selection of the signs of P_1 and P_2, the pressures which make the objective particle move in the same direction have been given the same sign.

Now, we try to derive an equation by which the calculation can be worked out with the experimental data. Taking a general equation of a straight line, $P = a + bv_0$, where a corresponds to P_2 and may be put as constant and b is also a constant which must be determined from the experimental data. With the method of least square, the equations of condition may be obtained:

$$\frac{\delta}{\delta a}(P-a-bv_0)^2 = 0\,, \qquad \frac{\delta}{\delta b}(P-a-bv_0)^2 = 0\,,$$

therefore $\qquad P-a-bv_0 = 0\,, \qquad Pv_0-av_0-bv_0^2 = 0\,.$

From n numbers of observations, we have:

$$\sum_{i=1}^{n} P_i - na - b\sum_{i=1}^{n} v_{0i} = 0\,, \qquad \sum_{i=1}^{n} P_i v_{0i} - a\sum_{i=1}^{n} v_{0i} - b\sum_{i=1}^{n} v_{0i}^2 = 0\,.$$

From these equations, we get b:

$$b = \frac{\sum_{i=1}^{n} P_i \sum_{i=1}^{n} v_{0i} - n\sum_{i=1}^{n} v_{0i}P_i}{\left(\sum_{i=1}^{n} v_{0i}\right)^2 - n\sum_{i=1}^{n} v_{0i}^2}\,.$$

Thus if we have b, the viscosity η will be calculated with the equation (3).

$$\eta = \frac{R^2}{4l}\frac{P}{v_0} = \frac{R^2(P-a)}{4lv_0} = \frac{R^2}{4l}b = \frac{R^2}{4l}\,\frac{\sum_{i=1}^{n} P_i \sum_{i=1}^{n} v_{0i} - n\sum_{i=1}^{n} P_i v_{0i}}{\left(\sum_{i=1}^{n} v_{0i}\right)^2 - n\sum_{i=1}^{n} v_{0i}^2}\,.$$

III. Experimental Materials and Conditions of Measurement. The liquids used for the experiment were glycerine, acetone solution of cellulose acetate, the former being of the highest grade of Price make. The density of the sample used in the experiments (1) and (2), was $D_{4^\circ}^{30^\circ} = 1.2506$, and that in the experiment (3) was $D_{4^\circ}^{25^\circ} = 1.2532$. For the acetone solution of cellulose acetate, "Cellit L, low viscosity" was used. A certain quantity of this substance was dissolved in pure acetone and then made free from solid matters with a centrifuge. Then, to determine the concentration, a certain quantity was drawn and dried up under reduced pressure and the residue was weighed. The concentration was 15.90%. This solution was used in the experiments (4), (5), (6), and (7).

For the objective particles, carbon and carborundum particles were used. The carbon particles had been made by burning benzene, then washing, defatting with CS_2, and heating at about 300°C. The diameter of the carbon particles was ca. 1/10 mm. to 1/100 mm. The carborundum powder used was that on the market for use in grinding. The powder was made to suspend in water and, after a while, the middle layer of the suspension was collected. The powder thus obtained had

Table 3.

Exp. No.	1	2	3	4	5	6	7
Sample liquid	Glycerine No. 1	do.	Glycerine No. 2	Acetone solution of cellulose acetate	do.	do.	do.
Capillary tube	Tube No. 1 $R=$ 0.040 cm. $l=$9.96 cm.	do.	do.	do.	do.	Tube No. 2 $R=$ 0.0545 cm. $l=$9.91 cm.	Tube No. 3 $R=$ 0.0965 cm. $l=$9.94 cm.
Objective	Carbon particle	do.	do.	Carborundum particle	do.	do.	do.
Quantity of particles added	0.1 g. per 100 c.c.	do.	do.	0.2 g. per 100 c.c.	do.	do.	do.
Measuring temp.	At room temp.	do.	25.00°C. ±0.01	30.0°C. ±0.1	26.4°C. ±0.1	do.	do.
Room temp.	30.0°C.	do.	20.0°C.	25.0°C.	29.5°C.	29.5°C.	28.0°C.
Remark			In thermostat	In simple bath	do.	do.	do.
Table No.	4	5	6	7	8	9	10
p–v Diagram	Fig. 3	Fig. 4	Fig. 5	Fig. 6.	Fig. 7	Fig. 8	Fig. 9

the diameter of the order of 1/50 mm. to 1/500 mm. These particles were treated first with concentrated hydrochloric acid, then with dilute hydrochloric acid, separated from the liquid with a centrifuge, made alkaline with ammonia, and washed with pure water, the centrifuge always being resorted to separate the liquid. Other experimental conditions are tabulated in Table 3.

IV. Results of Measurement and Their Discussion. (1) *Effect of the Quantity of Particles on Viscosity.* The experiment relative to the point discussed in **Theoretical, III, (b),** that is, the effect of quantity of particles added on viscosity was carried out and the following results were obtained.

(i) With an ordinary capillary viscosimeter, and glycerine without the particles, the following numbers were obtained: temp. 25.00°C. ± 0.01 (in thermostat); time of flow, (1) 19 min. 34.1 sec., (2) 19 min. 35.7 sec., (3) 19 min. 35.9 sec., (4) 19 min. 35.5 sec., mean value 19 min. 35.3 sec.

(ii) In the same glycerine, the carbon particles were mixed at the rate of 0.1 g. in 100 c.c. of glycerine, and the time of flow was measured with the same viscosimeter. The following data were obtained: temp. 25.00°C. ± 0.01; time of flow (1) 19 min. 36.2 sec., (2) 19 min 33.9 sec., (3) 19 min. 35.0 sec., (4) 19 min. 37.7 sec., mean value 19 min. 35.7 sec.

The increase of the time of flow due to the addition of the particle was 0.4 sec. This is 0.04% against 1175.3 sec. of the time of flow. This shows that the effect is within the experimental error and is quite negligible.

(2) *The Result of Measurement of Viscosity by the Microscopic Method.* (i) *Measurement at Room Temperature.* This was a preliminary measurement, which was indicated as experiment (1) and (2), and the results are given in Table 4 and 5. As the mean value $\eta = 4.36$ (absolute unit) was obtained. With an Ostwald viscosimeter, using the same sample, we obtained $\eta = 4.54$ (absolute unit). This is the value obtained in the thermostat of 30°C. As accuracy was not maintained in point of temperature, the comparison of the data is, strictly speaking, unreasonable but the result shows that the present method of viscosity measurement is quite a practical one.

(ii) *Measurement in the Thermostat.* From the preliminary test, the practical value of the present method of measurement was ascertained. So, definitive measurements were carried out. To gain the accuracy possible, the measuring tube was put in the thermostat, and the manometer was read with a measuring microscope up to the order of 0.01 mm. The sample was glycerine No. 2. The results are shown in Table 6 as exp. (3). The observed value was $\eta^{25^\circ} = 5.93_3$ (absolute unit).

The viscosity of the liquid relative to water was measured with the same sample, in the same thermostat with an ordinary capillary viscosimeter and with the aid of the value of absolute viscosity of water, $\eta^{25^\circ} = 5.93_2$ (absolute unit) was obtained. In this case, the time of flow of water was 24.8 sec. and that of the sample liquid was 13120 sec., the density was 1.2532, and as density and absolute viscosity of water, the values $D^{25^\circ}_{4^\circ} = 0.99707$ and $\eta^{25^\circ}_{H_2O} = 0.008941$ respectively were used in the calculation.

The comparison of these two values of viscosity shows that coincidence is quite satisfactory. But, the coincidence at the second decimal place seems to be by chance, as the error of time measurement is to appear at this decimal place. As regards other factors, the errors will appear at the fourth decimal place. Therefore, it is not difficult to increase the accuracy of measurement by increasing the time of measurement. From this definitive measurement, it may be accepted that this new method of measurement gives quite satisfactory results in point of accuracy.

(iii) *Measurement of Viscosity of Acetone Solution of Cellulose Acetate.* With this new method, a measurement of absolute viscosity of acetone solution of cellulose acetate was carried out. The results of measurement are given in Table 7. (Exp. No. 4)

(3) *The Relation between the Inner Radius of the Capillary Tube and the Value of Viscosity Measured.* When the inner radius of the capillary part of measuring tube changes, the relation comes in as the power of square of the radius by equation (3). To verify this relation, experiments were carried out, with the same liquid and with the measuring tubes of different radius in capillary parts. These are exp. (5), (6), (7), and the results are shown in Table 8–11.

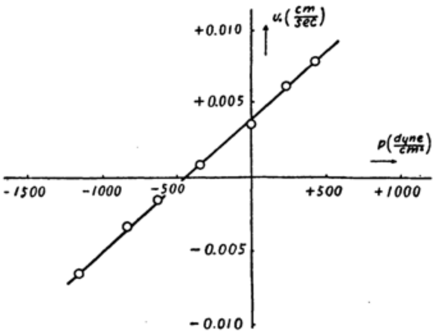


Fig. 3.

The sample was an acetone solution of cellulose acetate, same as stated in the preceding section. As the measurements were carried out in a simple bath, the results obtained are by no means perfect. However,

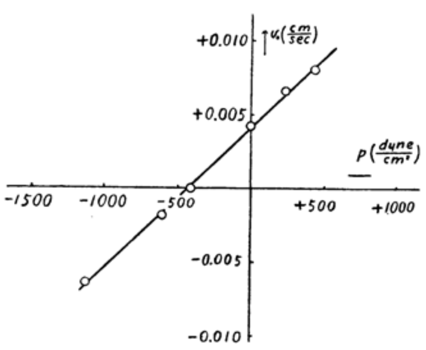
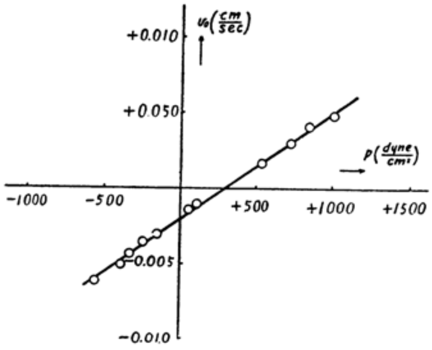


Fig. 4.



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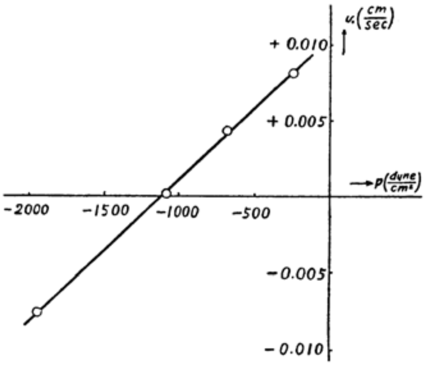

Fig. 6.

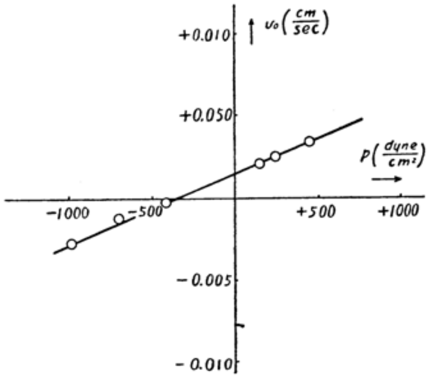

Fig. 7.

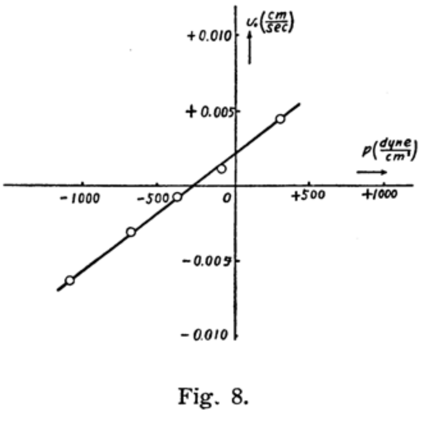

Fig. 8.

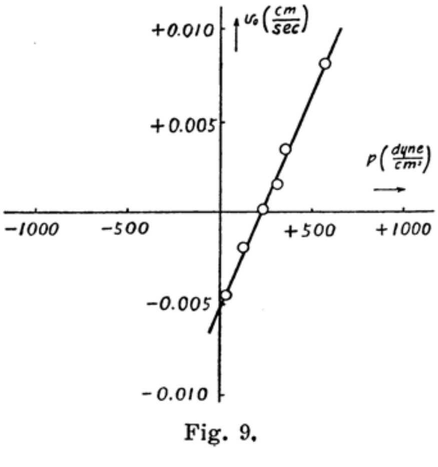

Fig. 9.

Table 4.

No.	$v_0\left(\frac{\text{cm.}}{\text{sec.}}\right)$	$P\left(\frac{\text{dyne}}{\text{cm.}^2}\right)$	v_0^2	Pv_0	Remark
1	−0.006413	−1133.0	0.00004113	+7.266	$\sum_{i=1}^{n} P_i \sum_{i=1}^{n} v_{0_i} = -15.73$
2	−0.001914	− 605.4	0.00000366	+1.159	$n\sum_{i=1}^{n} P_i v_{0_i} = +81.34$
3	−0.000161	− 410.1	0.00000003	+0.066	$\left(\sum_{i=1}^{n} v_{0_i}\right)^2 = +0.0001126$
4	+0.004275	± 0.000	0.00001828	±0.000	$n\sum_{i=1}^{n} v_{0_i}^2 = +0.001044$
5	+0.006678	+ 234.4	0.00004460	+1.565	$\frac{R^2}{4l} = 0.00004016$
6	+0.008144	+ 429.7	0.00006632	+3.500	$b = 1.043\times10^5$
$\sum_{i=1}^{n}$	+0.01061	−1484	+0.0001740	+13.56	$\eta = 4.18_7$
Viscosity (obs.)	1. By microscopic method				$\eta = 4.18_7$ (abs. unit)
	2. By capillary viscosimeter				$\eta = 4.54_4$ (abs. unit)

Table 5.

No.	$v^0\left(\frac{\text{cm.}}{\text{sec.}}\right)$	$P\left(\frac{\text{dyne}}{\text{cm.}^2}\right)$	v_0^2	Pv_0	Remark
1	+0.007895	+429.7	0.00006233	+3.393	$\sum_{i=1}^{n} P_i \sum_{i=1}^{n} v_{0_i} = -17.50$
2	+0.006256	+234.3	0.00003914	+1.466	$n\sum_{i=1}^{n} P_i v_{0_i} = +110.18$
3	+0.003620	±0.000	0.00001310	±0.000	$\left(\sum_{i=1}^{n} v_{0_i}\right)^2 = +0.00005664$
4	+0.000862	−351.5	0.00007430	−0.3030	$n\sum_{i=1}^{n} v_{0_i}^2 = +0.001187$
5	−0.001496	−624.9	0.00000224	+0.9348	$\frac{R^2}{4l} = 0.00004016$
6	−0.003268	−839.8	0.00001068	+2.732	$b = 1.130\times10^5$
7	−0.006413	− 1172	0.00004113	+7.516	$\eta = 4.53_8$
$\sum_{i=1}^{n}$	+0.007526	− 2324	0.0001695	+15.74	
Viscosity (obs.)	1. By microscopic method				$\eta = 4.53_8$ (abs. unit)
	2. By capillary viscosimeter				$\eta = 4.54_4$ (abs. unit)

Table 6.

No.	$v_0\left(\frac{\text{cm.}}{\text{sec.}}\right)$	$P\left(\frac{\text{dyne}}{\text{cm.}^2}\right)$	v_0^2	Pv_0	Remark
1	−0.001394	+59.71	0.00000194	−0.08384	$\sum_{i=1}^{n} P_i \sum_{i=1}^{n} v_{0_i} = -17.47$
2	−0.003000	−152.7	0.00000300	+0.4581	$n\sum_{i=1}^{n} P_i\, v_{0_i} = +214.35$
3	−0.003538	−243.7	0.00001252	+0.8623	$\left(\sum_{i=1}^{n} v_{0_i}\right)^2 = +0.0001184$
4	−0.004275	−331.8	0.00001828	+1.4185	$n\sum_{i=1}^{n} v_{0_i}^2 = +0.0016874$
5	−0.005130	−394.5	0.00002633	+2.0236	$\frac{R^2}{4l} = 0.00004016$
6	−0.006181	−560.9	0.00003820	+3.4666	$b = 1.478\times10^5$
7	−0.001016	+112.6	0.00000103	−0.1143	$\eta = 5.93_3$
8	+0.001676	+533.5	0.00000281	+0.8944	
9	+0.003054	+723.4	0.00000933	+2.2088	
10	+0.004171	+832.4	0.00001740	+3.4701	
11	+0.004750	+1028.0	0.00002256	+4.8820	
$\sum_{i=1}^{n}$	−0.010884	+1605.6	0.0001534	+19.486	
Visocity (obs.)	1. By microscopic method			$\eta = 5.93_3$ (abs. unit)	
	2. By capillary viscosimeter			$\eta = 5.93_2$ (abs. unit)	

Table 7.

No.	$v_0\left(\frac{\text{cm.}}{\text{sec.}}\right)$	$P\left(\frac{\text{dyne}}{\text{cm.}^2}\right)$	v_0^2	Pv_0	Remark
1	+0.008143	−254.2	0.00006631	−2.070	$\sum_{i=1}^{n} P_i \sum_{i=1}^{n} v_{0_i} = -7.919$
2	+0.004385	−684.5	0.00001928	−3.001	$n\sum_{i=1}^{n} P_i\, v_{0_i} = +76.80$
3	+0.0005109	− 1085	0.00000026	−0.5543	$\left(\sum_{i=1}^{n} v_{0_i}\right)^2 = +0.000002048$
4	−0.007600	− 1946	0.00005776	+14.79	$n\sum_{i=1}^{n} v_{0_i}^2 = +0.0007980$
5	−0.004008	− 1564	0.00001606	+6.196	$\frac{R^2}{4l} = 0.00004016$
$\sum_{i=1}^{n}$	+0.001431	− 5534	0.0001596	+15.36	$b = 1.064\times10^5$
Viscosity (obs.)	$\eta = 4.27_4$ (abs. unit)				

Table 8.

No.	$v_0\left(\frac{\text{cm.}}{\text{sec.}}\right)$	$P\left(\frac{\text{dyne}}{\text{cm.}^2}\right)$	v_0^2	Pv_0	Remark
1	+0.003466	+449.3	0.00001201	+1.5573	$\sum_{i=1}^{n} P_i \sum_{i=1}^{n} v_{0_i} = +32.70$
2	+0.002559	+253.9	0.00000256	+0.6497	$n\sum_{i=1}^{n} P_i\, v_{0_i} = +295.60$
3	+0.002129	+156.3	0.00000453	+0.3328	$\left(\sum_{i=1}^{n} v_{0_i}\right)^2 = +0.00005663$
4	−0.0002392	−420.0	0.00000006	+0.1005	$n\sum_{i=1}^{n} v_{0_i}^2 = +0.00151$
5	−0.001207	−713.0	0.00000146	+0.8606	$\frac{R^2}{4l} = 0.00004016$
6	−0.002788	−986.5	0.00000777	+2.7504	$b = 2.403\times10^5$
7	−0.01166	−3086	0.00013596	+35.983	
$\sum_{i=1}^{n}$	−0.007525	−4346	0.00016435	+42.23	
Viscosity (obs.)	$\eta = 9.65_4$ (abs. unit)				

Table 9.

No.	$v_0\left(\frac{\text{cm.}}{\text{sec.}}\right)$	$P\left(\frac{\text{dyne}}{\text{cm.}^2}\right)$	v_0^2	Pv_0	Remark
1	−0.006334	−1104.0	0.00004012	+6.993	$\sum_{i=1}^{n} P_i \sum_{i=1}^{n} v_{0_i} = +8.765$
2	−0.003109	−693.5	0.00000967	+2.156	$n\sum_{i=1}^{n} P_i\, v_{0_i} = +52.10$
3	−0.0006503	−390.7	0.00000042	+0.2541	$\left(\sum_{i=1}^{n} v_{0_i}\right)^2 = +0.00001934$
4	+0.001196	−97.67	0.00000143	−0.1168	$n\sum_{i=1}^{n} v_{0_i}^2 = +0.0003595$
5	+0.004500	+293.0	0.00002025	+1.1318	$\frac{R^2}{4l} = 0.00007593$
$\sum_{i=1}^{n}$	−0.004398	−1992.9	0.00007189	+10.42	$b = 1.274\times10^5$
Viscosity (obs.)	$\eta = 9.67_6$ (abs. unit)				

Table 10.

No.	$v_0\left(\frac{\text{cm.}}{\text{sec.}}\right)$	$P\left(\frac{\text{dyne}}{\text{cm.}^2}\right)$	v_0^2	Pv_0	Remark
1	−0.004499	+29.31	0.00002024	−0.1319	$\sum_{i=1}^{n} P_i \sum_{i=1}^{n} v_{0_i} = +11.036$
2	−0.001936	+136.8	0.00000375	−0.2649	$n\sum_{i=1}^{n} P_i v_{0_i} = +35.598$
3	+0.0001494	+234.5	0.00000002	+0.03503	$\left(\sum_{i=1}^{n} v_{0_i}\right)^2 = +0.00004629$
4	+0.001560	+302.9	0.00000243	+0.4725	$n\sum_{i=1}^{n} v_{0_i}^2 = +0.0006186$
5	+0.003514	+342.0	0.00001235	+1.202	$\frac{R^2}{4l} = 0.0002342$
6	+0.008016	+576.4	0.00006426	+4.620	$b = 4.222\times10^4$
$\sum_{i=1}^{n}$	+0.006804	+1622	0.0001031	+5.933	
Viscosity (obs.)	$\eta = 9.88_7$ (abs. unit)				

Table 11.

Tube No.	l (Length of tube)	R (Radius of tube)	R^2/l	b	$K\left(=\frac{R^2}{l}\times b\right)$
1	9.96	0.0400	0.0001606	2.403×10^5	38.6
2	9.91	0.0545	0.0003037	1.274×10^5	38.7
3	9.94	0.0965	0.0009368	4.222×10^4	39.6

as is shown in Table 11, the products of R^2/l and b are almost constant for all capillary tubes. This is what the theory requires. These results also indicate that the microscopic method of measurement is practical.

Summary.

(1) In connection with the method for measuring the viscosity of a liquid, a microscopic method, quite different from the methods hitherto known has been proposed and its possibility has been examined.

(2) The advantages of this microscopic method has been enumerated.

(3) The apparatus and the procedure of the measurement has been described.

(4) The theoretical relation of the influence of the fine particles added to the sample liquid on the viscosity of the liquid has been confirmed by experiment.

(5) The viscosity of glycerine and acetone solution of cellulose acetate has been measured by this new method.

(6) The results of measurements by the new method and those by the Ostwald viscosimeter have been compared, and the accuracy of the new method has been shown.

(7) It is shown that the relation between the viscosity and the radius of the measuring capillary tube is in accord with the theory.

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